The **bisection method** for approximating the real zeros of a continuous function is similar to the method used in Example 8. If you know that a zero exists in the closed interval $[a, b]$, the zero must lie in the interval $[a, (a+b)/2]$ or $[(a+b)/2, b]$. From the sign of $f([a+b]/2)$, you can determine which interval contains the zero. By repeatedly bisecting the interval, you can "close in" on the zero of the function.

TECHNOLOGY You can also use the *zoom* feature of a graphing utility to approximate the real zeros of a continuous function. By repeatedly zooming in on the point where the graph crosses the $x$-axis, and adjusting the $x$-axis scale, you can approximate the zero of the function to any desired accuracy. The zero of $x^3 + 2x - 1$ is approximately 0.453, as shown in Figure 1.38.

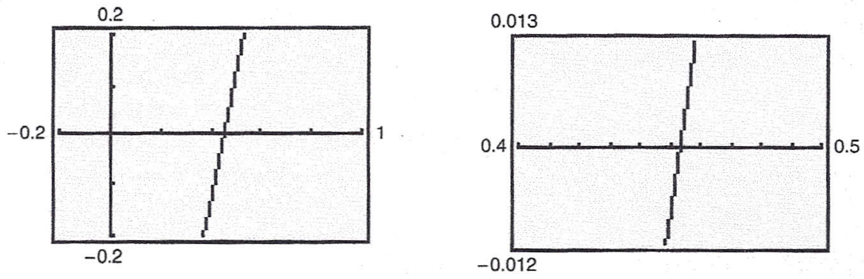


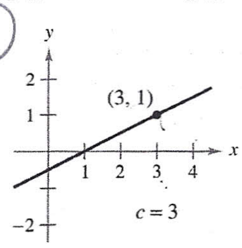

**Figure 1.38** Zooming in on the zero of $f(x) = x^3 + 2x - 1$

## EXERCISES FOR SECTION 1.4

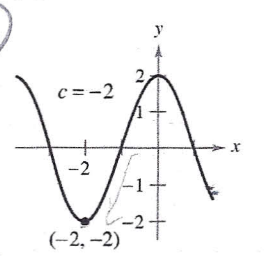
**In Exercises 1–6, use the graph to determine the limit, and discuss the continuity of the function.**

(a) $\lim\limits_{x\to c^+} f(x)$ (b) $\lim\limits_{x\to c^-} f(x)$ (c) $\lim\limits_{x\to c} f(x)$

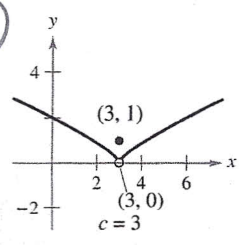
**1.**



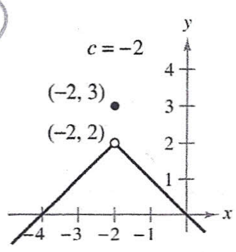

**2.**



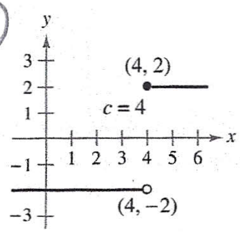

**3.**



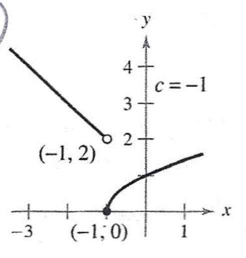

**4.**




**5.**




**6.**




**In Exercises 7–24, find the limit (if it exists). If it does not exist, explain why.**

**7.** $\lim\limits_{x\to 5^+} \dfrac{x-5}{x^2-25}$

**8.** $\lim\limits_{x\to 2^+} \dfrac{2-x}{x^2-4}$

**9.** $\lim\limits_{x\to -3^-} \dfrac{x}{\sqrt{x^2-9}}$

**10.** $\lim\limits_{x\to 4^-} \dfrac{\sqrt{x}-2}{x-4}$

**11.** $\lim\limits_{x\to 0^-} \dfrac{|x|}{x}$

**12.** $\lim\limits_{x\to 2^+} \dfrac{|x-2|}{x-2}$

**13.** $\lim\limits_{\Delta x\to 0} \dfrac{\dfrac{1}{x+\Delta x} - \dfrac{1}{x}}{\Delta x}$

**14.** $\lim\limits_{\Delta x\to 0^+} \dfrac{(x+\Delta x)^2 + x + \Delta x - (x^2 + x)}{\Delta x}$

**15.** $\lim\limits_{x\to 3^-} f(x)$, where $f(x) = \begin{cases} \dfrac{x+2}{2}, & x \le 3 \\ \dfrac{12-2x}{3}, & x > 3 \end{cases}$

**16.** $\lim\limits_{x\to 2} f(x)$, where $f(x) = \begin{cases} x^2 - 4x + 6, & x < 2 \\ -x^2 + 4x - 2, & x \ge 2 \end{cases}$

**17.** $\lim\limits_{x\to 1} f(x)$, where $f(x) = \begin{cases} x^3 + 1, & x < 1 \\ x + 1, & x \ge 1 \end{cases}$

**18.** $\lim\limits_{x\to 1^+} f(x)$, where $f(x) = \begin{cases} x, & x \le 1 \\ 1 - x, & x > 1 \end{cases}$

**19.** $\lim\limits_{x\to \pi} \cot x$

**20.** $\lim\limits_{x\to \pi/2} \sec x$

**21.** $\lim\limits_{x\to 4^-} (3[\![x]\!] - 5)$

**22.** $\lim\limits_{x\to 2^+} (2x - [\![x]\!])$

**23.** $\lim\limits_{x\to 3} (2 - [\![-x]\!])$

**24.** $\lim\limits_{x\to 1} \left(1 - \left[\!\!\left[-\dfrac{x}{2}\right]\!\!\right]\right)$